

B.Sc. Semester - 5 (NEP-2020) Examination

Oct/Nov - 2025

MATH: Abstract Algebra -1 & Complex Analysis -1 (Major-11)

Time: 2:00 Hours

Marks: 50

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 Answer any two out of three. (10)

- (1) Binary operation $*$ is defined on set G . if
 - (i) $*$ is associative in G .
 - (ii) for $\forall a \in G$ there exist $e \in G$ such that $a * e = a$
 - (iii) for $\forall a \in G$ there exist $b \in G$ such that $a * b = e$ then show that $(G, *)$ is group.
- (2) Prove that infinite cyclic groups has exactly two generators.
- (3) Find numbers of generators of cyclic group Z_{pq} . Where p and q are prime numbers.

Que.2 Answer any two out of three. (10)

- (1) Prove that product of disjoint cycles of symmetric group is commutative.
- (2) H and K be two Normal subgroups of group G . if $H \cap K = \{e\}$ then prove that $hk = kh$ for $\forall h \in H$ and $\forall k \in K$.
- (3) in usual notation $O(G/H) = 2$ then show that H is a normal subgroup of G . where G is group and H is subgroup of G .

Que.3 Answer any two out of three. (10)

- (1) In usual notations obtain Cauchy-Riemann conditions in Cartesian form.
- (2) Find an analytic function $f(z) = u + iv$ such that $u - v = e^{-x}(\sin y + \cos y)$
- (3) Define Harmonic function. Prove that $u = r^2 \cos 2\theta$ is a Harmonic function and find it's conjugate.

Que.4 Answer any two out of three. (10)

- (1) State and prove Cauchy's integral formula.
- (2) Obtain Laplace equation in polar form.
- (3) Find: $\int \frac{dz}{z - \frac{2}{z}}$; where $C: |z - 1| = 2$.

Que.5 Answer any two out of three. (10)

- (1) if $G = \{x \in \mathbb{C} / x^4 = 1\}$ then show that G is group under multiplication.
- (2) Define symmetric group and find number of elements of symmetric group S_n .
- (3) Prove that $\sin z$ is an analytic function.
